THE DEVELOPMENT OF A DESIGN PROCEDURE FOR TWO-WAY FLOOR SLABS

BY

CHESTER PAUL SIESS

B.S., Louisiana State University, 1936
M.S., University of Illinois, 1939

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN
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This thesis was prepared under the direction of N. M. Newmark, Research Professor of Structural Engineering, who suggested the problem and the method of attack. His valued help and guidance in all phases of the work are gratefully acknowledged.

Credit is also due to Professor Newmark for two basic features of the distribution procedure: the use of total or average moments rather than maximum moments, and the application of the Hardy Cross moment distribution concept to the problem of continuous plates.

R. UNIV. BIBLIOTHEEK LEIDEN

INTRODUCTION

The chief object of this thesis was the development of a design procedure for two-way reinforced concrete floor slabs in buildings. This was accomplished in three steps: (1) An approximate distribution procedure was developed for the calculation of moments in uniformly loaded elastic plates continuous over rigid supports. (2) This procedure was used to determine the moments in continuous slabs for various assumed conditions of loading and edge restraint. And (3), modifications were made to the moments thus obtained to account for certain effects beyond the scope of the distribution procedure. The resulting moments were used as a basis for the proposed design procedure for two-way slabs.

No tests of two-way slabs were made in connection with this thesis. Consideration was given, however, to available test data in the development of the design procedure.

DISTRIBUTION PROCEDURE

The distribution procedure is applicable to the calculation of moments in rectangular elastic plates supported on all four sides and continuous in two directions. Only loads distributed uniformly over an entire panel may be considered, but all panels need not be loaded. The torsional rigidity of the supporting beams may be taken into account, but it is assumed in all cases that the beams do not deflect. It is further assumed that Poisson's ratio is equal to zero.

The procedure is analogous to the Hardy Cross moment distribution method for continuous beams and frames; that is, fixed-edge moments are calculated, unbalanced moments are distributed in proportion to the relative stiffnesses of the elements of the structure, and portions of the distributed moments on each edge are carried over to the other edges. The distribution procedure developed for plates is approximate, however, and in this respect differs importantly from the Cross method. The moments considered are the average moments acting on the edge of a panel, and are functions of the ratio of sides of the panel as well as the span length and loading. The stiffness factors for the slab are functions of the span length, the depth of the slab, and the ratio of sides, while the carry-over factors are functions of the ratio of sides only. The effect of the edge moments on the positive moments in the interior of a simply-supported slab are determined approximately by means

of correction factors which are functions only of the ratio of sides of the panel.

Numerical values of the various constants are given in the thesis for panels having ratios of sides from 0.5 to 1.0. The average fixed-edge moments and the average positive moments in the simply-supported slab are exact, but the stiffness and carry-over factors and the positive moment correction factors are approximate. These distribution constants were derived by means of a semi-rational procedure based on the exact moments in isolated rectangular plates supported on all sides and having various combinations of fixed and simply-supported edges.

The accuracy of the distribution was determined by applying it to the analysis of certain continuous slabs for which the correct moments were known. The error in the edge moments computed from the approximate analysis was less than 5 per cent except for two cases in which the moments were quite small. The approximate positive moments varied from 8 per cent less to 17 per cent greater than the exact values.

MOMENTS IN CONTINUOUS SLABS

As an intermediate step to the development of a design procedure for two-way slabs the distribution procedure was used for the calculation of moments in a number of continuous slabs. The several variables studied were: (1) the location of a panel with respect to a discontinuous edge; (2) the amount of torsional restraint offered by the supporting beams; (3) various types of loading, including all panels loaded, and two types of partial or "live" loading; (4) the ratio of sides of the panels, for slabs in which all panels were similar; and (5) combinations of panels of various span lengths and ratios of sides. Eight slabs consisting of 25 equal panels arranged in five rows of five panels each and one slab consisting of 16 unequal panels were analyzed for various loading conditions. A total of 83 different analyses were made.

Studies of a few actual designs indicated the ratio of the torsional stiffness of a beam to the flexural stiffness of the slab for typical two-way slabs varied between 1.2 and 2.3. The values of this ratio used in these studies were 0, 1.0, and 2.0. It was concluded that a value of 1.0 should be used when computing moments in interior panels but that a value of 2.0 should be used when computing negative moments at a discontinuous edge.

In addition to the dead-load condition of uniform load in all panels, two types of partial loading were considered. One of these was the familiar maximum moment "checkerboard" loading. For positive moment in the interior of a panel and for negative moment at the outer edge of an edge panel, alternate panels are loaded in checkerboard fashion. For maximum negative moment over an interior beam, the checkerboard pattern is applied on either side of, and symmetrical about, a line passing along the edge in question. The other type is referred to as "single-panel" loading and consists simply of loading only the one panel in question for positive moment or moment at an exterior edge. For negative moment over an interior beam, only the two adjacent panels are loaded. Although, in general, somewhat greater moments are produced by the checkerboard pattern, it was decided on the basis of the analyses described above and on the basis of probability of occurrence that the single-panel type of loading would be used for the calculation of live-load moments in the development of the design procedure.

It was further concluded as a result of these studies that if torsional rigidity of the beams is considered the moments produced in a continuous slab as a result of either checkerboard or single-panel loadings are practically the same for all panels, regardless of their location with respect to an edge. For uniform loadings, however, somewhat larger differences were obtained between the moments in corner, edge, and interior panels.

DEVELOPMENT OF DESIGN PROCEDURE FOR TWO-WAY SLABS

The actual development of the design procedure required that certain modifications be made in the moments determined by means of the distribution procedure for typical panels of continuous slabs on rigid beams. First, it was necessary to take account of the effect of deflection of the beams on the moments in the slab. This was done in an approximate manner after certain assumptions had been made regarding the relative stiffness of the beam and slab. The next step involved the replacement of the separate coefficients for dead-load and live-load moments by a combined coefficient which could be applied to the sum of the dead and live loads. After comparison of several values, a live load-dead load ratio of 3.0 was chosen as the basis for this combination.

The final modification of the slab moments consisted of a blanket reduction of 20 per cent in all of the maximum moments considered. This reduction was made in recognition of the redistribution of moment between the various elements of the structure which occurs as maximum loads are approached. The moments produced when all panels are loaded, and such redistribution is thus not possible, were considered as a limiting case in making this reduction in the maximum moments produced by partial loadings.

Somewhat less extensive studies were made of the distribution of moment across the width of the slab, and of the moments for which the supporting beams should be designed, in order to complete the development of the design procedure.

PROPOSED DESIGN PROCEDURE FOR TWO-WAY SLABS

The proposed design procedure is intended to apply to reinforced concrete slabs consisting of rectangular panels continuous in two directions and supported on all four sides by reinforced concrete beams built monolithically with the slab, or by steel beams encased in concrete. The slab should contain three or more rows of panels in each direction, and the ratio of width to length of any panel should not be less than 0.5. The ratio of live load to dead load assumed in the derivation of the moment coefficients was 3.0, but the procedure may be used with safety and economy for values of this ratio varying from 1.0 to 6.0. The procedure is applicable only to those cases in which the live load is assumed to be distributed uniformly over the entire area of each loaded panel.

The principal features of the design procedure are presented below in a form patterned after that of the 1940 Joint Committee Report, Articles 809 through 815. The principal differences between the proposed procedure and that specified in the Joint Committee Report are: the specification of average moments, differences in the magnitude of the moment to be provided for, and the use of the same design moments in interior, edge, and corner panels.

Design Sections: The critical section for negative moment should be taken along the edges of the panel at the face of the supporting beams, and the critical section for positive moment should be taken along the center lines of the panels.

For placement of the reinforcement, the width of the slab at each critical section should be divided into a middle strip, covering the central half of the width, and two column strips, covering the outer quarters.

For bending up truss or bent bars used for positive moment, and for the calculation of anchorage requirements for straight bars similarly used, the lines of contraflexure for positive moment should be assumed to be at a distance of $\frac{1}{6}$ the shorter span from each edge of the panel. For computing anchorage requirements for negative moment reinforcement, the lines of contraflexure should be assumed to be at a distance of $\frac{1}{5}$ the shorter span from each edge of the panel.

Bending Moment Coefficients: Coefficients for average bending moments for various ratios of sides, b/a, are given in the table.

MOMENT COEFFICIENTS IN TERMS OF wb^2

Moment	Span	Ratio of Sides, b/a						
		0	0.5	0.6	0.7	0.8	0.9	1.0
Negative Moment over Interior Beams	Short	0.0780	0.0548	0.0501	0.0455	0.0408	0.0362	0.0315
	Long	0.0390	0.0390	0.0387	0.0378	0.0363	0.0342	0.0315
Positive Moment	Short	0.0472	0.0304	0.0270	0.0236	0.0202	0.0169	0.0135
	Long	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135	0.0135

These coefficients when multiplied by wb^2 give the average bending moment per unit of width of slab. The quantity w is the sum of the dead and live load per unit of area, and b is the length of the shorter span. The values given in the table apply to all panels, whether located in the interior, at an edge, or at a corner. Negative moments at a discontinuous edge should be taken as two-thirds the moments over an interior beam.

In the middle strip, a moment equal to 1.25 times the average moment should be used, and in the column strips, a moment equal to 0.75 times the average should be used.

Distribution of Unequal Negative Moment at Supports: If adjacent panels have different dimensions or loadings the use of the specified moment coefficients will result in different values of the negative moments on either side of a supporting beam. When this condition exists, some modification of the moments should be made

based on the relative rigidity of the slabs and the torsional restraint offered by the beams. It is assumed that the supporting beams offer a restraint equivalent to the average of the stiffnesses of the adjacent slabs and, on this basis, two-thirds of the unbalanced negative moment should be distributed to the two spans in proportion to their stiffnesses. If the thickness of the slab is the same for the two panels, and if their span lengths do not differ by more than 25 per cent, it will usually be satisfactory to assume that their stiffnesses are equal.

Moments and Shears in Beams: The loads on the supporting beams may be assumed as the uniformly distributed load within the tributary areas bounded by the intersection of 45-degree lines from the corners with the median line of the panel parallel to the long side.

Moments and shears in the beams should be computed for 80 per cent of the above loading, using the coefficients for maximum moments and shears given in the A.C.I. Building Code (ACI 318-41), Article 701 (c).

Shear in Slab: Shearing stresses in the slab should be computed on the assumption that the load is distributed to the supporting beams in the manner described in the preceding section, and no reduction should be applied.

APPENDICES

Supplementary data in the form of exact solutions for moments in various types of slabs are given in three appendices to the thesis.

Solutions are given in Appendix A for uniformly loaded rectangular plates supported on all four sides and having various combinations of fixed and simply-supported edges.

Appendix B contains exact solutions for moments in uniformly loaded continuous slabs containing 9 to 15 rectangular panels of various sizes and shapes.

Appendix C contains the results of exact solutions for moments in rectangular plates with concentrated loads and with either three or four edges fixed.

VITA

Chester Paul Siess was born in Alexandria, Louisiana, July 28, 1916. Upon graduation from Bolton High School of that city he entered Louisiana State University, and received the degree of Bachelor of Science in Civil Engineering in June 1936.

He was employed by the Louisiana Highway Commission, chiefly as Soils Engineer in the Testing Laboratory, until September 1937. He then received an appointment as Special Research Graduate Assistant in Theoretical and Applied Mechanics at the University of Illinois where he worked half-time on a cooperative investigation of reinforced concrete highway bridges, and received the Master of Science degree in Civil Engineering in June 1939.

He was employed from June 1939 to April 1941 in the Soil Testing Laboratory of the Department of Subways and Superhighways, Chicago, Illinois; and from April to September 1941 in the bridge division of the Office of Chief Engineer, New York Central System, in Chicago.

In September 1941 he was appointed Special Research Associate in Theoretical and Applied Mechanics at the University of Illinois, with assignment to the cooperative investigation of reinforced concrete highway bridges. In September 1945 he was made Special Research Assistant Professor, which position he now holds.

He is a member of the following organizations: American Concrete Institute; American Society for Testing Materials; Associate Member, American Society of Civil Engineers; Sigma Xi; Tau Beta Pi; Phi Kappa Phi; Omicron Delta Kappa; and Phi Eta Sigma.

He is co-author of Bulletins Nos. 336, 363, and 375 of the University of Illinois Engineering Experiment Station, all dealing with the work of the investigation of concrete highway bridges.